This exam has 20 multiple choice questions worth 5 points each. Fill in your answer card as described on the cover page of this exam. You are NOT allowed a calculator or note card of any kind on this test.

Good Luck.

1. The period of $\sin(\pi t/3)$ is:

   (A) $\pi$  (B) $2\pi$  (C) $6$  (D) $\pi/3$  (E) $3$  (F) $2\pi/3$  (G) none of these

For Problems 2-5, let $f(t) = 1 + \frac{\cos(\pi t)}{2} + \frac{\cos(2\pi t)}{4} + \frac{\cos(3\pi t)}{8} + \frac{\cos(4\pi t)}{16} + ...$

2. The minimal period of $f$ is:

   (A) $\pi$
   (B) $2\pi$
   (C) $4\pi$
   (D) $1$
   (E) $2$
   (F) $\pi/2$
   (G) $4$
   (H) $1/\pi^2$

3. $f$ is: (A) Even  (B) Odd  (C) Neither  (D) Both  (E) Impossible to tell if it is even or odd.
4. Let \( x_p \) be a periodic solution of \( x'' + (\omega_n)^2 x = f(t) \). For which \( \omega_n \) is the amplitude of the \( \cos(\pi t) \) term in the Fourier Series of the solution the greatest?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 9
(F) 16
(G) None of the above

5. The differential equation in problem 3 above does not have a periodic solutions if \( \omega_n = ? \)

(A) only \( \pi \) (that is, it has periodic solutions for any other value of \( \omega_n \))
(B) 1
(C) only \( \pi/4 \) (that is, it has periodic solutions for any other value of \( \omega_n \))
(D) 1/2
(E) 2
(F) either \( \pi/2 \) or \( 3\pi \) (among other numbers, and not exclusive or)
(G) either 1 or 2 (among other numbers, and not exclusive or)
6. Let $f$ be the periodizaton (of period $2\pi$) of the function that is $-1$ on the interval $[-\pi, 0)$ and $1$ on the interval $(0, \pi)$. Which of the following statements best describes the Fourier series of $f$? (Make sure to choose the one that conveys the most amount of information about the Fourier coefficients of $f$.)

(A) $a_k = 0$ for all $0 \leq k$
(B) $a_k = 0$ for all $0 < k$
(C) $b_k = 0$ for all $0 \leq k$
(D) $b_k = 0$ for all positive even $k$
(E) $b_k = 0$ for all positive odd $k$
(F) $a_k = 0$ for all positive even $k$ and $b_k = 0$ for all positive odd $k$.
(G) $b_k = 0$ for all $0 \leq k$ and $a_k = 0$ for all positive even $k$
(H) $b_k = 0$ for all $0 \leq k$ and $a_k = 0$ for all positive odd $k$
(I) $a_k = 0$ for all $0 \leq k$ and $b_k = 0$ positive even $k$
(J) $a_k = 0$ for all $0 \leq k$ and $b_k = 0$ positive odd $k$
7. Let \( f(t) = 3 \cos t \). Below is the graph of a function \( g(t) \).

Which of the following functions is \( g(t) \)?

(A) \( u(t)f(t) \)
(B) \( u(t+2)f(t+2) \)
(C) \( u(t-2)f(t+2) \)
(D) \( u(t+2)f(t-2) \)
(E) \( u(t-2)f(t-2) \)
(F) \( u(t)f(3t) \)
(G) \( u(t+2)f(3t+2) \)
(H) \( u(t-2)f(3t+2) \)
(I) \( u(t+2)f(3t-2) \)
(J) \( u(t-2)f(3t-2) \)
(K) None of the above
8. Suppose \( f(t) = 1 \) when \( t \geq 1 \) and when \( t < 1 \), \( f(t) \) is defined in each of the following ways:

i. \( f(t) = t \)

ii. \( f(t) = 2t \)

iii. \( f(t) = \frac{1}{t-1} \)

Which of the resulting functions is regular on the interval \([0,2]\)?

(A) i

(B) ii

(C) iii

(D) i and ii

(E) i and iii

(F) ii and iii

(G) i and ii and iii

(H) None of them
9. Let \( f(t) = \sin 3t \) and \( g(t) = 1 \). Find the convolution product \((f \ast g)(t)\).

(A) \( t \)
(B) \( -t \)
(C) \( \frac{1}{3} \cos(3t) \)
(D) \( -\frac{1}{3} \cos(3t) \)
(E) \( \frac{1}{3} (1 - \cos(3t)) \)
(F) \( -\frac{1}{3} (1 - \cos(3t)) \)
(G) \( \cos(3t) \)
(H) \( \frac{1}{3} \sin(3t) \)
(I) None of the above
On the next two problems let: \( f(t) = 0 \) for \( t < 0 \), \( f(t) = 3t - 1 \) for \( 0 < t < 1 \), \( f(t) = 0 \) for \( t > 1 \).

10. \( f(t) = \)

   (A) \( u(t) - u(t - 1) \)

   (B) \( u(t) - u(t - 1) \)(3t)

   (C) \( u(t)(3t - 1) \)

   (D) \( u(t) - u(t - 1) \)(3t - 1)

   (E) \( u(t) - u(t - 1) \)(t)

   (F) \( u(t) - u(t - 1) \)(3t - 1) + \( \delta(t) \)

   (G) \( u(t) - u(t - 1) \)(3t - 1) + \( \delta(t) + \delta(t - 1) \)

   (H) \( u(t) - u(t - 1) \)(3t - 1)\( \delta(t) \)

11. \( f'(t) = \)

   (A) \( 3(u(t) - u(t - 1)) - \delta(t) + \delta(t - 1) \)

   (B) \( 3(u(t) - u(t - 1)) \)

   (C) \( u(t) - u(t - 1) \) - \( \delta(t) - 2\delta(t - 1) \)

   (D) \( 3(u(t) - u(t - 1)) - \delta(t) - 2\delta(t - 1) \)

   (E) \( \delta(t) - \delta(t - 1) \)

   (F) \( \delta(t) + \delta(t - 1) \)

   (G) \( \delta(t) - 3\delta(t - 1) \)

   (H) \( 3(u(t) - u(t - 1)) - \delta(t) + 2\delta(t - 1) \)
12. What is the unit impulse response for the LTI operator $2D^2 + 4D + 4I$?

(A) $\frac{1}{2} u(t)e^{-t}\cos t$
(B) $\frac{1}{2} e^{-t}\sin t$
(C) $\frac{1}{2} u(t)e^{t}\sin t$
(D) $u(t)e^{-t}\sin t$
(E) $u(t)e^{-t}\cos t$
(F) $\frac{1}{2} u(t)e^{-t}\sin t$
(G) $\frac{1}{2} \delta(t)e^{-t}\sin t$
(H) $\frac{1}{2} u(t)e^{-t}\sin \omega$
13. What is the unit step response for the operator in problem 12 above?

(A) \( \frac{1}{4} u(t)(1 - e^{-t}(cost + sint)) \)

(B) \( u(t)(1 - e^{-t}(cost + sint)) \)

(C) \( (1 - e^{-t}(cost + sint)) \)

(D) \( \frac{1}{4} u(t)(1 - (cost + sint)) \)

(E) \( \frac{1}{4} u(t)(1 - e^{-t}sint) \)

(F) \( \frac{1}{4} u(t)(1 - e^{-t}cost) \)

(G) \( \frac{1}{2} u(t)(1 - e^{-t}(cost + sint)) \)
14. Let \( p(D) \) be the operator whose unit impulse response is given by \( w(t) = e^{-t} \). Which of the following gives the solution to \( p(D) = u(t) \) with rest initial conditions. (\( u(t) \) is the unit step function.)

(A) \( \int_0^t e^{-\tau - \xi} \, d\tau \)

(B) \( \int_0^\infty e^{\xi - \tau} \, d\tau \)

(C) \( e^{-\xi} \)

(D) \( \int_0^\infty e^{-\xi} \, d\tau \)

(E) \( \int_0^\infty e^{-\xi} \, d\tau \)

(F) \( \int_0^\infty e^{-\xi} \, d\tau \)

(G) \( e^{-\xi + \xi} \)

(H) None of the above
15. Find \( L^{-1}\left\{\frac{1}{s(\sqrt{s^2 + 1})}\right\} \).

A) \( t^2 - t \)

B) \( t - t^2 \)

C) \( 1 - \frac{t^2}{4} + \frac{t^4}{64} \)

D) \( 1 - \frac{t^2}{4} \)

E) \( \cos t \)

F) \( \cos(\ln t) \)

G) \( 1 - \cos t \)

H) \( \frac{1}{3} e^{-t} \sin 3t \)

I) \( \frac{1}{3} e^{-t} \cos 3t \)

J) None of the above
16. Find \( L^{-1} \left\{ \frac{1}{s^2 + 2s + 10} \right\} \).

A) \( t^2 - t \)

B) \( t - t^3 \)

C) \( 1 - \frac{t^2}{4} + \frac{t^4}{64} \)

D) \( 1 - \frac{t^2}{4} \)

E) \( \cos t \)

F) \( \cos(\ln t) \)

G) \( 1 - \cos t \)

H) \( \frac{1}{3} e^{-t} \sin 3t \)

I) \( \frac{1}{3} e^{-t} \cos 3t \)

J) None of the above
17. Suppose \( f(t) \) has the properties: \( f(0) = 0, \ f'(0) = 0 \), and the Laplace transform

\[
L\{f(t)\} = \frac{1}{s^2 + s + 1}.
\]

Find \( L\{f''(t)\} \).

A) \( \frac{3}{s^2 + s + 1} \)

B) \( \frac{s^2}{s^2 + s + 1} \)

C) \( \frac{3}{(s^2 + 2s^2 + 9)} \)

D) \( \frac{s}{(s^2 + 3s^2 + 9)} \)

E) \( \frac{e^{-s}}{s^3} \)

F) \( \frac{6e^{-s}}{s^4} \)

G) \( \frac{1}{s} \)

H) \( \frac{1 - e^{-s}}{s^2 (s^2 + 1)} \)

I) \( \frac{e^{-s}}{s^2 (s^2 + 1)} \)

J) None of the above
18. Use the Laplace transform to turn the initial value problem into an algebraic equation in $X(s)$.

$$x'' + 8x' + 15x = e^t \quad x(0) = 1, x'(0) = 2$$

(a) $(s^2 + 8s)X(s) = \frac{1}{s - 1} - 15$

(b) $X(s) = (s^2 + 8s + 15)(\frac{1}{s - 1} + 2s + 9)$

(c) $(X(s) + 3)(X(s) + 5) = \frac{1}{s - 1}$

(d) $s^2 + 8s + 15X(s) = \frac{1}{s - 1}$

(e) $(s^2 + 8s + 15)X(s) = s + 10$

(f) $(s^2 + 8s + 15)X(s) = 2s + 9$

(g) $(s^2 + 8s + 15)X(s) = 2s + 9 + \frac{1}{s - 1}$

(h) $(s^2 + 8s + 15)X(s) = \frac{1}{s - 1} + s + 10$

(i) $(s^2 + 8s + 15)X(s) = \frac{1}{s - 1}$

(j) $(s^2 + 8s + 15)X(s) = \frac{1}{s + 1}$

(k) None of the above
19. Use the Laplace transform to turn the initial value problem into an algebraic equation in \( X(s) \).
Solve for \( X(s) \).
\[
2x'' - 5x' - 12x = \cos t \quad x(0) = x'(0) = 0
\]

Find the Laplace transform of the solution.

(a) \( X(s) = \frac{s}{s^2 + 1} \)

(b) \( X(s) = \frac{s}{(s^2 + 1)(s - 4)(2s + 3)} \)

(c) \( X(s) = \frac{1}{(s^2 + 1)(s - 4)(2s + 3)} \)

(d) \( X(s) = \frac{s - 1}{(s - 4)(2s + 3)} \)

(e) \( X(s) = \frac{1}{s^2 + 1} \)

(f) \( X(s) = \frac{(s - 4)(2s + 3)}{s^2 + 1} \)

(g) \( X(s) = \frac{1}{(s - 4)(2s + 3)} \)

(h) \( X(s) = \frac{s}{(s - 4)(2s + 3)} \)

(i) \( X(s) = \frac{s(s - 4)(2s + 3)}{s^2 + 1} \)

(j) None of the above
20. Let \( y(t) \) be the solution of the I.V.P. \( y'' + 2y = \sin 3t, \ y(0) = y'(0) = 0. \) Find 
\( Y(s) = L[y(t)]. \)

A) \( \frac{3}{s^3 + s + 1} \)

B) \( \frac{s^2}{s^3 + s + 1} \)

C) \( \frac{3}{(s^2 + 2)(s^2 + 9)} \)

D) \( \frac{s}{(s^2 + 3)(s^2 + 9)} \)

E) \( \frac{e^{-s}}{s^3} \)

F) \( \frac{6e^{-s}}{s^3} \)

G) \( \frac{1}{s} \)

H) \( \frac{1 - e^{-s}}{s^2(s^2 + 1)} \)

I) \( \frac{e^{-s}}{s^2(s^2 + 1)} \)

J) None of the above